# Engineering Notes

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## Effects of Nonconstant Enthalpy Addition on Fan-Nozzle Combinations

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### Nomenclature

$A$ = area
$H$ = stagnation enthalpy
$h$ = enthalpy
$M$ = Mach number
$P$ = static pressure
$T_{RA}$ = net thrust with variable fan blade enthalpy input
$T_{RR}$ = net thrust with constant fan blade enthalpy input
$T_R$ = ratio of net thrust, $T_{RA}/T_{RR}$
$u$ = axial velocity
$x$ = axial coordinate
$\gamma$ = specific heat
$\epsilon$ = constant
$\eta$ = fan efficiency
$\rho$ = density
$\tau_r$ = $1+(\gamma-1/2)M_0^2$
$\tau_c$ = stagnation temperature ratio across the fan constant enthalpy case

*Subscripts*

$1$ = flow station (see Fig. 1)
$2$ = flow station (see Fig. 1)
$c$ = reference constant enthalpy case
$e$ = flow station (see Fig. 1)
$0$ = flow station (see Fig. 1)
$t$ = stagnation conditions

### Introduction

COMPRESSOR or turbine stage loading may be increased, or the efficiency improved for a given loading, through a nonconstant enthalpy addition to the flow along the length of the blade.[1] The resultant nonuniform flow at exit from the stage will have losses if the flow is passed through a nozzle for propulsive purposes. Two limiting cases are considered in estimating the expected effect of flow nonuniformities on engine thrust. For an engine with nonconstant enthalpy addition across the fan stage, but with otherwise perfect components, the flow can be 1) completely unmixed (isentropic), or 2) fully mixed in an ideal constant area mixer before expansion through the nozzle.

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### Analysis

It is assumed that the blade loading is such that $H$ is given by

$$H_1/H_c = 1+(\epsilon/2)(2x-1) \tag{1}$$

This enthalpy distribution is an often used reference form.[2-4]

#### A. Isentropic Flow through the Nozzle

For this case

$$\int_0^1 u_e \mathrm{d}x = \sqrt{2H_c}\int_0^1 \left\{1+\frac{\epsilon}{2}(2x-1)-\frac{h_e}{H_c}\right\}^{1/2}\mathrm{d}x = \sqrt{2H_c}\left(\frac{2}{3\epsilon}\right)\left(\left\{1-\frac{h_e}{H_c}+\frac{\epsilon}{2}\right\}^{3/2} - \left\{1-\frac{h_e}{H_c}-\frac{\epsilon}{2}\right\}^{3/2}\right) \tag{2}$$

Assuming ideal flow and exit pressure equal to ambient pressure for this case, it follows that

$$\frac{\int_0^1 u_e \mathrm{d}x}{\int_0^1 u_{ec}\mathrm{d}x} = \frac{2}{3\epsilon}\left(1-\frac{1}{\tau_r\tau_c}\right)^{-1/2}\left[\left(1-\frac{1}{\tau_r\tau_c}+\frac{\epsilon}{2}\right)^{3/2} - \left(1-\frac{1}{\tau_r\tau_c}-\frac{\epsilon}{2}\right)^{3/2}\right] \tag{3}$$

Denoting the flight velocity as $u_0$, routine cycle analysis gives

$$(u_{ec}/u_0) = [(\tau_r\tau_c - 1)/(\tau_r - 1)]^{1/2} \tag{4}$$

$$T_R = \frac{(u_{ec}/u_0)\left(\int_0^1 u_e \mathrm{d}x \Big/ \int_0^1 u_{ec}\mathrm{d}x\right) - 1}{(u_{ec}/u_0) - 1} \tag{5}$$

Since $\epsilon$ must be small, a binomial expansion leads to the approximate form

$$T_R \simeq 1 - \frac{\epsilon^2}{96}\frac{(\tau_r\tau_c)^2}{(\tau_r\tau_c-1)^{3/2}}\frac{1}{(\sqrt{\tau_r\tau_c - 1}-\sqrt{\tau_r - 1})} \tag{6}$$

#### B. Fully Mixed Flow

A momentum balance across the frictionless constant area mixer gives

$$\int (P+\rho u^2)_1 \mathrm{d}A = (P+\rho u^2)_2 A \tag{7}$$

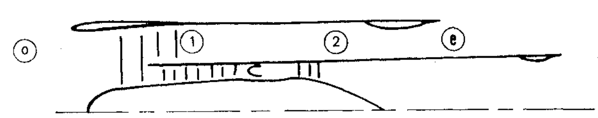


Fig. 1 Definition of engine reference.

Conservation of stagnation enthalpy gives $H_2 = H_c$. At station 1, both the entropy and pressure are constant; hence the static temperature is constant. Therefore, the continuity equation and Eq. (7) lead to

$$\frac{\gamma}{(H_c/H_1)}\int_0^1\left(\frac{1}{\gamma M_1}+M_1\right)\mathrm{d}x = \frac{(1+\gamma M_2^2)}{M_2(1+\frac{\gamma-1}{2}M_2^2)^{1/2}} \equiv \phi^{1/2} \tag{8}$$

Evaluation of the left side of this equation (and hence of the function $\phi$) leads to a quadratic equation for $M_2^2$. Thus, noting that

$$M_1^2 = \frac{2}{\gamma-1}\left(\frac{H_1}{h_1}-1\right) \tag{9}$$

Eq. (1) leads to

$$M_1^2 = \frac{2}{\gamma-1}\left(\frac{H_c}{h_1}\left\{1+\frac{\epsilon}{2}(2x-1)\right\}-1\right) \tag{10}$$

It may be noted from Eq. (10) that the maximum imaginable value of $\epsilon$ would be that leading to $M_1 = 0$ on the hub ($x=0$), so that

$$\epsilon_{\max} = \frac{2(H_c/h_1)-1)}{H_c/h_1} = \frac{(\gamma-1)M_c^2}{1+\frac{\gamma-1}{2}M_c^2}$$

With Eq. (10), the function $\phi$ is easily integrated to give

$$\phi = \left\{\frac{1}{\epsilon}\left(\frac{H_c}{h_1}\right)^{-3/2}\left[\sqrt{2(\gamma-1)}\,(\beta_+ - \beta_-) + \frac{2\gamma}{3}\frac{2}{\gamma-1}(\beta_+{}^3 - \beta_-{}^3)\right]\right\}^2 \tag{11}$$

in which

$$\beta_\pm = \left\{\frac{H_c-h_1}{h_1} \pm \frac{H_c}{h_1}\frac{\epsilon}{2}\right\}^{1/2}$$

Solution of Eq. (8) then gives

$$M_2^2 = 2[\phi - 2\gamma + \{\phi^2 - 2(\gamma+1)\phi\}^{1/2}]^{-1} \tag{12}$$

The continuity equation may be used to determine the static pressure at station 2 by writing

$$A_1 = \rho_2 u_2 A_2 \int_0^1 \frac{\mathrm{d}x}{\rho_1 u_1} = A_2$$

from which

$$\frac{P_2}{P_1} = \frac{h_2}{h_1}\frac{1}{M_2\int_0^1 \frac{\mathrm{d}x}{M_1}}, \tag{13}$$

The integral is easily evaluated using Eq. (9). These equations may be summarized in a form allowing sequential solution as follows: $\epsilon$, $M_c$, $\tau_r$, $\tau_c$, and $\gamma$ would be prescribed.

## Summary

$$\frac{H_c}{h_1} = 1 + \frac{\gamma-1}{2}M_c^2$$

$$\beta\pm = \left\{\frac{H_c}{h_1} - 1 \pm \frac{H_c}{h_1}\frac{\epsilon}{2}\right\}^{1/2}$$

$$\phi = \left(\frac{1}{\epsilon}\left(\frac{H_c}{h_1}\right)^{-3/2}\left\{\sqrt{2(\gamma-1)}\,(\beta_+ - \beta_-) + \frac{2\gamma}{3}\frac{2}{\gamma-1} \times (\beta_+^3 - \beta_-^3)\right\}\right)^2$$

$$M_2^2 = 2[\phi - 2\gamma + \{\phi^2 - 2(\gamma+1)\phi\}^{1/2}]^{-1}$$

$$\frac{h_2}{h_1} = \frac{1+\frac{\gamma-1}{2}M_c^2}{1+\frac{\gamma-1}{2}M_2^2}$$

$$\frac{P_2}{P_1} = \frac{h_2}{h_1}\frac{M_c}{M_2}\left(\frac{H_c-h_1}{h_1}\right)^{-1/2}\frac{\beta_+ + \beta_-}{2}$$

$$\frac{P_{t2}}{P_{tc}} = \frac{P_2}{P_1}\left(\frac{h_2}{h_1}\right)^{-\frac{\gamma}{\gamma-1}}$$

$$\frac{\int_0^1 u_e \mathrm{d}x}{u_{ec}} = \left(1-\frac{1}{\tau_r\tau_c}\right)^{-1/2}\left(1-\frac{1}{\tau_r\tau_c}\left(\frac{P_{t2}}{P_{tc}}\right)^{-\frac{\gamma-1}{\gamma}}\right)^{1/2}$$

$$\frac{u_{ec}}{u_0} = \left\{\frac{\tau_r\tau_c - 1}{\tau_r - 1}\right\}^{1/2}$$

$$T_R = \frac{(u_{ec}/u_0)\left(\int_0^1 u_e \mathrm{d}x/u_0/u_{ec}\right) - 1}{(u_{ec}/u_0) - 1}$$

Though unwieldy in appearance, these equations are easily programmed on a desk calculator. After a great deal of algebra, using binomial expansions, the following very simple approximate form is obtained:

$$T_R = 1 - [\sqrt{\tau_r\tau_c - 1}(\sqrt{\tau_r\tau_c - 1} - \sqrt{\tau_r - 1})]^{-1}\frac{\left(1+\frac{\gamma-1}{2}M_c^2\right)^2}{48(\gamma-1)M_c^2}\epsilon^2 \tag{14}$$

Numerical verification indicates that both Eqs. (6) and (14) are quite accurate. The largest fractional error in the perturbation terms found in the range of values considered was only about 10%.

In order to compare the losses just predicted with a possible thrust gain due to compressor efficiency improvement, the net thrust of an otherwise perfect engine with a nonperfect compressor efficiency was obtained. Again, using a binomial expansion (in terms of $\Delta\eta_c = 1 - \eta_c$), it follows that

$$T_R = 1 - \frac{\Delta\eta_c}{2}\frac{\tau_c - 1}{\tau_c}[\sqrt{\tau_r\tau_c - 1}(\sqrt{\tau_r\tau_c - 1} - \sqrt{\tau_r - 1}]^{-1} \tag{15}$$

Simple relationships now follow for the "breakeven" efficiency improvement required if the fan efficiency improvement resulting from the nonconstant enthalpy addition is to overcome the losses introduced. Thus, Eqs. (6, 14, and 15) give

$$(\Delta\eta_c)\text{ break even} = \frac{\tau_c}{\tau_c - 1}\frac{(\tau_r\tau_c)^2}{\tau_r\tau_c - 1}\frac{\epsilon^2}{48} \qquad \text{(nonmixing)}$$

$$(\Delta\eta_c)\text{ break even} = \frac{\tau_c}{\tau_c - 1}\frac{(1+\frac{\gamma-1}{2}M_c^2)^2}{(\gamma-1)M_c^2}\frac{\epsilon^2}{24}, \qquad \text{(mixing)}$$

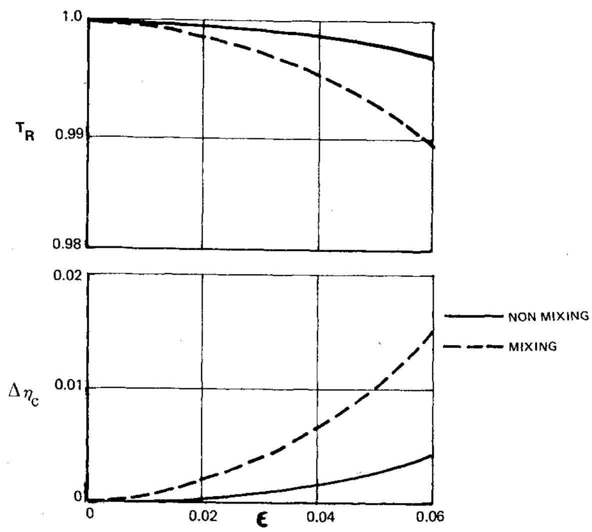


**Fig. 2 Effects of fan exit profiles on nozzle thrust and breakeven fan efficiency.**

## Results

Sample results were calculated for the case where the compressure pressure ratio is 1.5, $M_0 = 0.85$, and $M_c = 0.5$.

The reduction in net thrust of the fanstream vs $\epsilon$ is shown in Fig. 2. It can be seen that the mixing losses introduce a substantial penalty. It is also evident that substantial increases in compressor efficiency must be achieved to overcome the losses due to nonconstant enthalpy distribution if substantial mixing occurs. The quadratic nature of the losses due to nonconstant enthalpy make the concept of using slight variations in enthalpy quite attractive.

Although these calculations are based on the simplified model of a perfect engine, it is expected that the predicted tendencies for the ratio of the net thrusts will remain quite accurate.

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# Estimating Aircraft True Airspeed Using Temperatures from Two Different Probes

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## I. Introduction

THE National Hail Research Experiment (NHRE) has used aircraft extensively to obtain meteorological measurements around and inside hailstorms. During an interesting multiaircraft thunderstorm research investigation, the electronic signal from the dynamic pressure sensor on one aircraft was "lost," rendering the meteorological data useless. The following describes a procedure employed to estimate the missing true airspeed (TAS) in order to correct the raw meteorological measurements.

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## II. Concept and Equations

The TAS of an aircraft can be determined by measuring the pitot-static pressure, $\Delta P = P_0 - P_s$ ($P_0$ is total pressure representing combined dynamic and static effects), the static pressure $P_s$, and the total air temperature $T_0$; and applying Bernoulli's equation in the form:

$$(V_1{}^2/2) + C_p T_1 = (V_2{}^2/2) + C_p T_2 \tag{1}$$

where $C_p$ is the specific heat at constant pressure. At the stagnation point of the pitot tube, airflow relative to the aircraft is zero, so that $V_2 \equiv 0$, $T_2 \equiv T_0$ = total air temperature; $V_1 \equiv V_a$ = aircraft TAS; $T_1 \equiv T$ = ambient air temperature; and the TAS may be expressed as:

$$V_a{}^2 = 2C_p(T_0 - T) \tag{2}$$

In reality, the dynamic effect on an aircraft temperature-sensing element is not a purely adiabatic process, and the measured temperature is less than $T_0$. The ratio of heating actually experienced by the sensor to the adiabatic heating is referred to as $r$, the recovery coefficient of the probe, and the practical form of Eq. (2) becomes

$$T_p = T + r(V_a{}^2/2C_p) \tag{3}$$

where $T_p$ is the temperature measured by a specific probe. According to Eq. (3), the temperature sensed by a probe is a function of the airspeed. Similarly, the difference between temperatures sensed by two probes having different recovery factors is a function of TAS and, conversely, TAS is proportional to the difference in probe temperatures. By designating the temperature measured by two different probes as $T_{p_1}$ and $T_{p_2}$, we may write

$$T_{p_1} - r_1(V_a{}^2/2C_p) = T_{p_2} - r_2(V_a{}^2/2C_p) \tag{4}$$

and, after rearrangement,

$$V_a{}^2 = [(T_{p_2} - T_{p_1})/(r_2 - r_1)]2C_p \tag{5}$$

Fortunately, the aircraft was equipped with two different temperature sensors: a Rosemount Model 102, deiced configuration b, total temperature probe[1] and a probe of a reverse flow type.[2] The recovery factors for these probes, as mounted on the aircraft, have been determined experimentally by a series of aircraft speed runs to be:

$r_1 = r_{rf} = 0.6425$ (reverse flow)

$r_2 = r_{rm} = 0.972$ (Rosemount model 102)

By defining $T_{p_1} \equiv T_{rf}$ and $T_{p_2} \equiv T_{rm}$, Eq. (5) can be rewritten as

$$\mathrm{TAS}_c \equiv V_a = 78.0973\,(T_{rm} - T_{rf})^{1/2} \tag{6}$$

and can be used to calculate the missing true airspeed.

Due to probe temperature changes associated with sensor wetting, the technique of calculating TAS by Eq. (6) is not applicable to in-cloud flight segments.[3] The calculation is also sensitive to temperature measurement errors and requires very stable temperature sensors, signal conditioning, and recording components. For example, the TAS of the aircraft involved during normal thunderstorm research flights would be within